

Evolution and anti-evolution in a minimal stock market model

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Abstract

We present a novel microscopic stock market model consisting of a large number of random agents modeling traders in a market. Each agent is characterized by a set of parameters that serve to make iterated predictions of two successive returns. The future price is determined according to the offer and the demand of all agents. The system evolves by redistributing the capital among the agents in each trading cycle. Without noise the dynamics of this system is nearly regular and thereby *fails* to reproduce the stochastic return fluctuations observed in real markets. However, when in each cycle a small amount of noise is introduced we find the typical features of real financial time series like fat-tails of the return distribution and large temporal correlations in the volatility without significant correlations in the price returns. Introducing the noise by an evolutionary process leads to different scalings of the return distributions that depend on the definition of fitness. Because our realistic model has only very few parameters, and the results appear to be robust with respect to the noise level and the number of agents we expect that our framework may serve as new paradigm for modeling self generated return fluctuations in markets.

Key words: Econophysics; Agent-based model; Stock market model; Evolution; Fat tails

1 Introduction

Empirical studies of stock markets and foreign exchange rates demonstrate that financial time series exhibit some universal characteristics [1,2,3,4,5].

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The distribution of returns usually has fat-tails and there are large temporal correlations in the volatility (“volatility clusters”) without correlation in the returns. These findings are in contrast to the hypothesis that, due to independence of market traders financial time series are simple random walks [6]. This raises the question which underlying dynamics is responsible for these properties.

Microscopic models developed in recent years [7,8,9,10,11,12] attempted to reproduce the essential features of real stock market time series. Although these models reproduced some statistical aspects of price time series, a deeper understanding of the underlying processes still remains difficult, mainly, because most of the models are not very robust with respect to the choice of parameters and some even depend on the number of agents in the system [13].

Starting from a different motivation, game theoretical models [14,15,16,17] were considered to understand the interplay between different players in an artificial market. These models have only a few parameters and allow an analytical understanding of interaction in multi-agent-models. The simplicity of these models, however makes an application to real markets difficult.

In this paper we present a simple paradigmatic model which combines these two approaches. In particular we apply a game theoretical Ansatz with only a very small number of free parameters and a price determination close to reality [18]. This is a necessary step towards understanding the mechanisms that lead to fat-tails and volatility clusters and may enable the analysis of the dynamics. In this paper we concentrate on the effect that different evolution strategies have on the statistical properties of price fluctuations.

In section 2 we describe the basic model. In section 3 we show that our model naturally reproduces the fat-tails of return distributions and the emergence of volatility clusters. In section 4 we introduce evolution as a source of noise into the model and finally show how different selection mechanisms influence the shape of the return distributions.

2 Basic model

In our model of a stock market N agents trade by swapping stocks into cash and vice versa. Initially every agent i receives a number of S_0 stocks and an amount M_0 of cash. The decision of one agent to buy or to sell stocks is determined by a set of P parameters $\alpha_{\Delta t}^i(\Delta t = 0, \dots, \tau)$. These parameters define the linear prediction model of each agent and are randomly drawn from a normal distribution with mean 0 and variance σ^2 ($\mathcal{N}(0, \sigma)$).

Every simulation cycle of the model consists of three steps: In the first step, the agents make a prognosis of two successive future returns based on their individual prediction model. In the second step the overall demand and supply of the stocks is calculated and the new price is determined according to an order book. In the third step all possible orders are executed and the strategies of all agents are rearranged by a perturbation of their parameters.

2.1 Prediction of the future returns

For the prediction of the future price each agent only takes the log-returns $\ln(\frac{p(t-1)}{p(t-2)}) = r(t-1)$ of the price history $p(t)$ into account, which ensures that the absolute value of a stock price is not relevant.

For simplicity we assume that every agent makes only a linear prediction of the two following future returns by weighting the past $P = \tau + 1$ returns of the time series with her individual parameters α . At time $t - 1$ an agent predicts the returns $\hat{r}(t)$ and $\hat{r}(t + 1)$ using the following equations:

$$\hat{r}_t^i = f^i(r_{t-1}, r_{t-2}, \dots, r_{t-\tau}) = \alpha_0^i + \sum_{\Delta t=1}^{\tau} \alpha_{\Delta t}^i r(t - \Delta t)$$

and

$$\hat{r}_{t+1}^i = f^i(\hat{r}_t^i, r_{t-1}, \dots, r_{t-\tau+1}) = \alpha_0^i + \alpha_1^i \hat{r}_t^i + \sum_{\Delta t=2}^{\tau} \alpha_{\Delta t}^i r(t - \Delta t + 1)$$

For both successive predictions the agents use the same deterministic linear prediction function f^i , clearly using for the second prediction the outcome of the first, i.e. the strategy of an agent i is fully determined by the P prediction parameters $(\alpha_0^i, \alpha_1^i, \alpha_2^i, \dots, \alpha_{\tau}^i)$.

2.2 Determination of the new price

After all agents made their predictions the decision whether an agent sells or buys shares is made: If $\hat{r}_{t+1}^i < 0$ the agent i makes an offer at the price $\hat{p}_t^i = p_{t-1} e^{\hat{r}_t^i}$ and becomes a potential seller for this trading cycle. Otherwise, i.e. if $\hat{r}_{t+1}^i > 0$, the agent i makes a bid at price $\hat{p}_t^i = p_{t-1} e^{\hat{r}_t^i}$ and intends to become a buyer.

This means that if e.g. the agent believes that the price will go down from

time t to time $t + 1$ he will try to sell his stocks, as long as the price at time t is higher than \hat{p}_t^i .

Now two functions are computed to fix the new price: an offer function with all offers and a demand function containing all bids (see also [18]).

$$O(p) = \sum_{i: \hat{r}_{t+1}^i < 0} S^i \Theta(p - \hat{p}_t^i)$$

$$D(p) = \sum_{i: \hat{r}_{t+1}^i > 0} \Delta S^i \Theta(\hat{p}_t^i - p)$$

S^i is the total number of stocks agent i owns. $\Delta S^i = \text{int}[\frac{M^i}{\hat{p}_t^i}]$, is the integer number of stocks, which agent i is able to buy with her money M^i . $\Theta(x)$ is the Heavy-side function with $\Theta(x) = 1$ for $x \geq 0$ and $\Theta(x) = 0$ for $x < 0$. $O(p)$ is the offer function, where all offers ($\hat{r}_{t+1}^i < 0$) are collected. The function is monotonously rising with steps at the limit prices, where an agent i is willing to sell his stocks (if he had at least one). $D(p)$ is the respective monotonously decreasing demand function (see Fig.(1)) As seen from the equation above every agent either intends to sell all her stocks or to use all her money which means that the agents fully believe in their prognosis. Both functions together represent the order book of our stock market model. Now we calculate the minimum of both functions at some price p :

$$Z(p) = \min\{O(p), D(p)\}$$

This functions reflects the transaction volume in stocks at a certain price p , i.e. it represents the turnover function.

In order to determine the new price we take the minimum and the maximum argument of $Z(p)$ at the interval of the maximum turnover: $p_{min} = \min(\arg\max Z(p))$ and $p_{max} = \max(\arg\max Z(p))$. The new price is then defined by the weighted mean between these two points: $p(t) = \frac{p_{min}O(p_{min}) + p_{max}D(p_{max})}{O(p_{min}) + D(p_{max})}$. Sometimes it happens that there is a spread between the highest bid and the lowest offer, therefore the turnover function equals zero for all values of p . Due to the fact that the standard process of price determination lead to no meaningful price in these situation we alternatively choose the price in the middle between the highest bid and the lowest offer.

2.3 Execution of orders and agent dynamics

When the new price is fixed, the agents execute their orders. All buyers with $\hat{p} < p_{min}$ buy ΔS^i stocks and all sellers with $\hat{p} > p_{max}$ sell all their stocks S^i . At the point of intersection the offer in general does not match the bid and therefore here only the difference between offer and bid can be traded.

Before the next cycle is started we add a small amount of noise of amplitude $\tilde{\sigma}$ to all parameters of the agents. With ξ drawn from a normal distribution ($\mathcal{N}(0, 1)$) the parameters of each agent then are perturbed to yield:

$$\alpha_{\Delta t}^i(t+1) = \frac{\sigma}{\sqrt{\sigma^2 + \tilde{\sigma}^2}}(\alpha_{\Delta t}^i + \tilde{\sigma}\xi)$$

The factor $\frac{\sigma}{\sqrt{\sigma^2 + \tilde{\sigma}^2}}$ has been introduced to keep the variance of the parameter distribution independent of the choice of $\tilde{\sigma}$ at the constant value σ^2 . This ensures that simulations with different $\tilde{\sigma}$ can be compared.

3 Results of the basic model

The results of the basic model are essentially influenced only by the noise added to the parameters in each cycle. In order to understand the role of this noise we examined the effect of it's amplitude $\tilde{\sigma}$. For $\tilde{\sigma} = 0$, all agents keep their prediction parameters fixed all the time leading to a quite regular time series (Fig.2), which reflects the deterministic dynamics for the price determination. For most of the randomly chosen initial conditions, the average values of $r(t)$ are close to zero, while occasionally it is larger than the amplitude of fluctuations.

While in the complete absence of noise the model behaves incompatible to the dynamics of real stock markets, a small amount of noise is sufficient to reproduce typical features of real stock markets (Fig.3). The simulation clearly exhibits the phenomenon of volatility clustering (Fig.3b) and therefore induces correlations in the absolute log-returns of the time series (dashed line in Fig.3c). The correlations of the raw returns show only a small anti-correlation (solid line in Fig.3c), which is far too small to be responsible for the correlations in the absolute returns. Furthermore we see a non-Gaussian shape of the return distribution (Fig.3d) with fat tails that vanish at longer time scales (Fig.3e). The power spectrum (Fig.3f) of the price exhibits scaling with an exponent slightly below two, implying that the Hurst exponent $H \lesssim \frac{1}{2}$.

For larger $\tilde{\sigma}$ the parameters of the agents are perturbed stronger during each time step which leads to a smooth change between fat-tailed and Gaussian distributions (Fig. 4). This shows that the memory in the parameters and the capital of the agents is a necessary requirement for fat-tails in our model. If the noise on these parameters is too strong the trading behavior of the agents becomes random and we finally observe the expected Gaussian distribution for $\tilde{\sigma}$. We found that the noise level sufficient to yield the behavior described above depends on the number of agents - the more agents the less noise keeps the system from becoming regular. Also, the power law behavior breaks down if $P \leq 2$ - for all simulations with $P \geq 3$ we saw complex dynamics similar to the results shown above.

4 Evolution as noise source

Here we discuss a simple variant of our model, which can be seen in analogy to an evolutionary process. Instead of perturbing the parameters of all agents as described in section 2.3 we in each cycle choose one agent which we eliminate and introduce a new agent with new random parameters into the market. All other parts of the basic model remain unchanged.

We consider three different strategies: 1. Eliminate the poorest, 2. the richest or 3. a random one. 1. and 2. are based on the capital an agent owns ($C^i(t) = M^i(t) + p(t)S^i(t)$) and represent an 'evolutionary' and an 'anti-evolutionary' mechanism of selection. The new agent starts with money M_0 and shares S_0 and receives new prediction parameters α^i randomly drawn from a Gaussian distribution ($\mathcal{N}(0, \sigma)$). The noise introduced by this evolutionary mechanism is different from the basic model in two respects: Firstly, in a model with a larger number of agents this procedure corresponds to a smaller relative noise level in the system. Second, we now not only have noise in the parameter space, but instead also introduce noise in the money and the stocks.

At the end of each trading period the amount of stocks and cash are both normalized by $\sum_{i=1}^N M^i = NM_0$ and $\sum_{i=1}^N S^i = NS_0$ to remain comparable to the basic model. This could be interpreted as some sort of tax that each agent has to pay for letting the new agent enter the market.

5 Results of the evolutionary models

Figs.5a,d show the result for the model in which the poorest trader is replaced, Figs.5b,e for the model in which a random agent is chosen, and Figs.5c,f depicts the effects of each time selecting the richest agent. Surprisingly, all

these evolutionary versions of our model lead to non-Gaussian fat-tailed return distributions. In particular the tails in Fig.5f are larger than in Figs.5b,d. Comparing the correlations of these processes, we see that a selection of the poorest agent kills trends in the time series faster and leads to a smaller autocorrelation than in the case with selecting the richest which exhibits quite strong anti-correlations. In order to check the robustness of our results we investigated the effect of the total number of agents on the results. As discussed above larger markets employing an evolutionary principle imply a smaller relative noise level at each cycle. We find that also with increasing number of agents the return distributions show a strongly non-Gaussian shape.

6 Summary and Discussion

We presented a simple paradigmatic model for a stock market by combining game theoretical approaches with a realistic mechanism for price determination that reproduces the basic features of returns in real price time series. It consists of traders which compete by making predictions of future returns. In order to be consistent with real trading we found that a two step prediction is necessary which we modeled by iterating a linear predictor for each trader. To determine the parameters for prediction we used two different approaches: In our basic model we chose our parameters from a Gaussian distribution and change it every time step by adding some noise to the parameters. In the evolutionary versions we used different kinds of evolution to choose agents and replaced them by new random ones.

Both models qualitatively reproduce important statistical features of returns in real price time series like volatility clustering and fat-tails using only a few parameters: the number of agents N , the initial amount of stocks and cash, the complexity P characterizing the agents, and the mean and the standard deviation of the parameter distribution. In the basic model, the level of an intrinsic Gaussian noise has to be specified additionally. As long as this parameter is not too small the scaling appears to be realistic (see Fig.4), while below a critical level, the scaling behavior broke down. However, our results indicate that in the limit of $N \rightarrow \infty$ the amount of noise, necessary for realistic scaling, vanishes. When $\tilde{\sigma}$ is increased the distribution of returns changes via a power law to a Gaussian distribution. This shows that the volatility clusters are not caused by the mechanism determining the price, but by the memory contained in the parameters and, most notably, in the capital of the agents. In contrast, the noise induced by the evolutionary mechanisms needs no adjustment. In some cases the scaling of the return distribution became even more algebraic when we decreased the effective noise level by increasing the number of agents. The evolutionary models enable to study how the dynamics of the agents in parameter space influence the statistics of the time series.

To this end we modeled three different kinds of evolution. Our study demonstrates that strong evolutionary pressure to perform well in a market leads to a destruction of correlations in the time series, while random selection and, in particular, anti-evolution induce larger (anti-)correlations. Remarkably, we found particularly clear power laws for anti-evolution.

In all variants of our model the results did not critically depend on the number of agents N , and for large N the scalings rather improved. To our knowledge, our approach therefore represents the first solution to a long standing problem in modeling markets [19]. Therefore, our model could be considered as a prototype of a self organized system, which tends to evolve toward a critical state, as one would also expect from real stock markets. In particular, we think that the combination of a price mechanism with two consecutive return predictions and an evolution mechanism that works nearly without stochasticity will lead to a deeper understanding of the underlying dynamics of stock markets.

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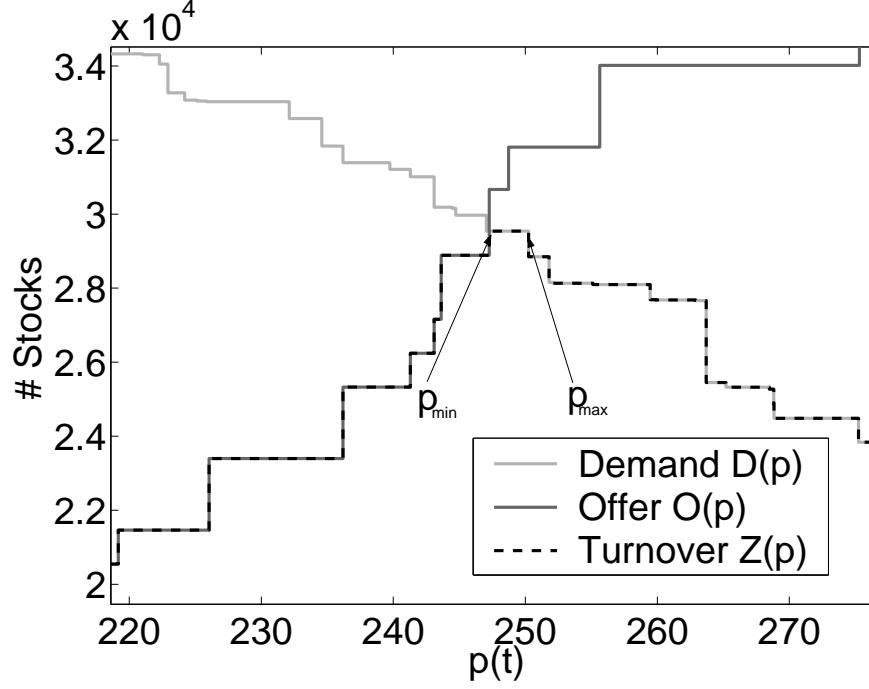


Fig. 1. The demand $D(p)$, offer $O(p)$ and turnover $Z(p) = \min\{D(p), O(p)\}$ as function of the price p in one trading cycle. The turnover reflects how many shares would be traded at a certain price. The new price is determined in the interval between p_{min} and p_{max} where the function $Z(p)$ has its maximum.

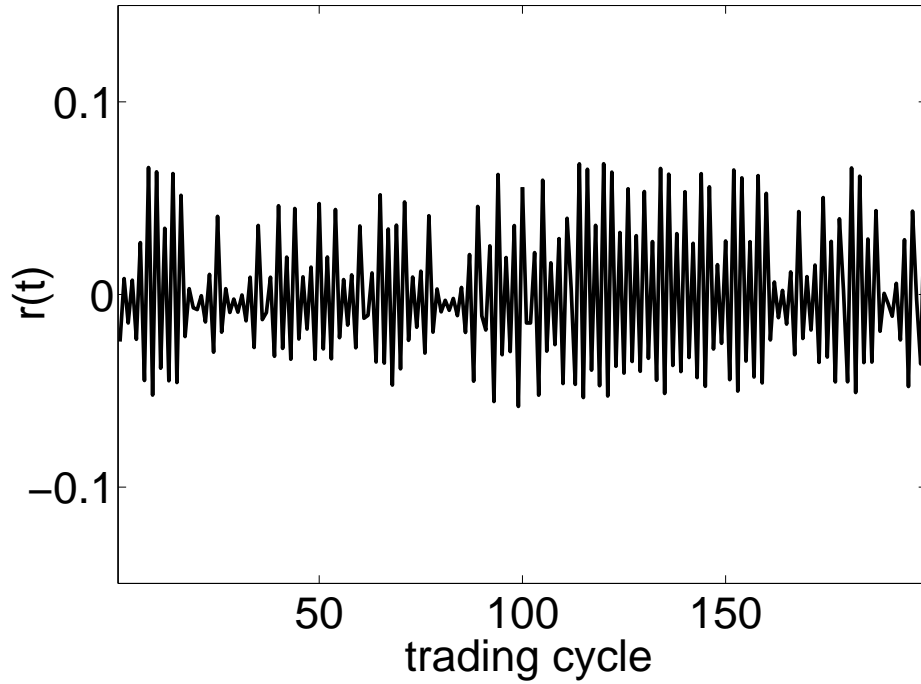


Fig. 2. Log-returns $r(t) = \log(\frac{p(t)}{p(t-1)})$ of a simulation produced by the Basic Model with parameters $N = 1000, P = 3, S(0) = 1000, M(0) = 100000, \sigma = 1, \tilde{\sigma} = 0$.

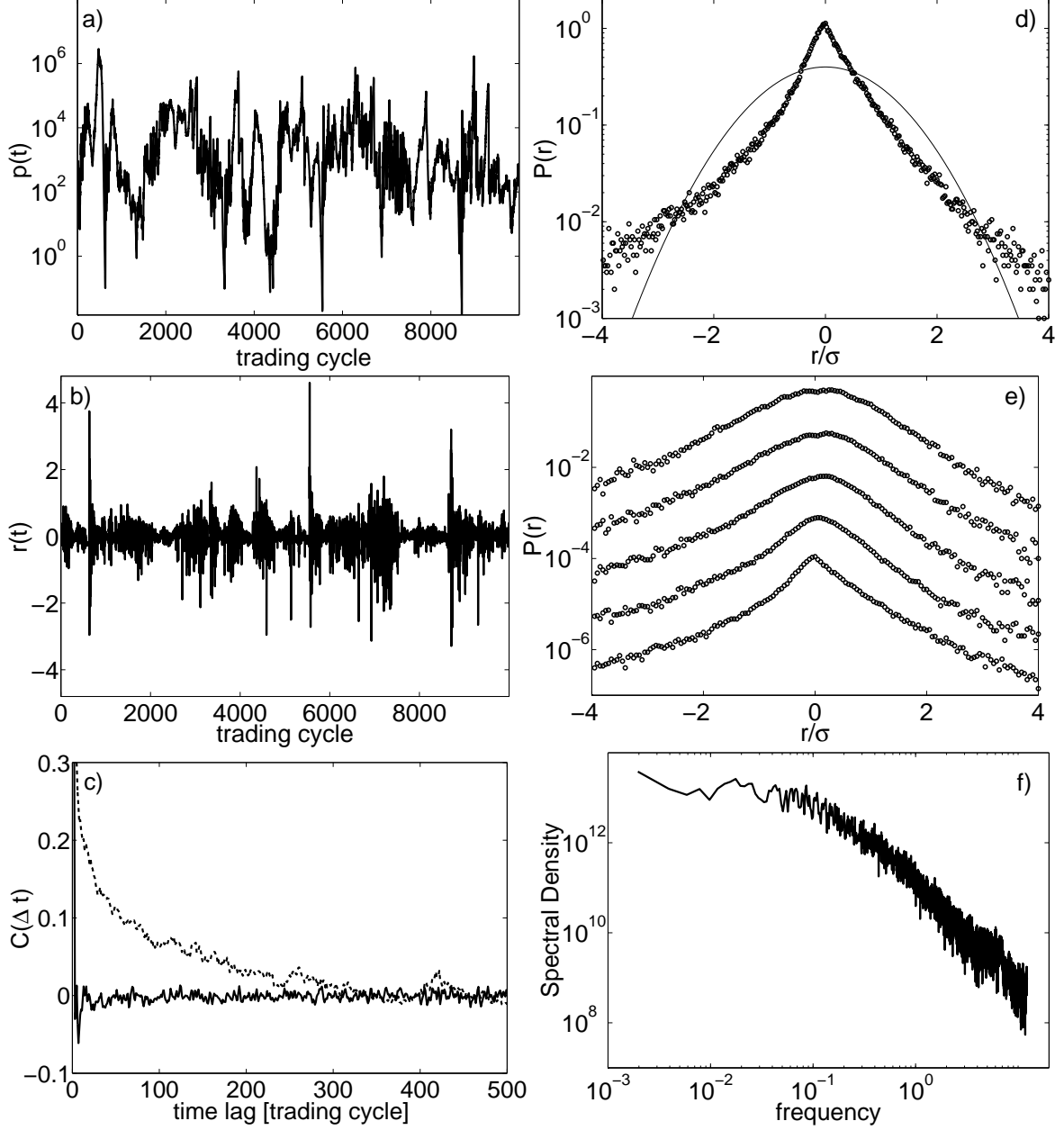


Fig. 3. Simulation results obtained from the Basic Model with parameters $N = 1000, P = 3, S(0) = 1000, M(0) = 100000, \sigma = 1, \tilde{\sigma} = 0.01$. A minimum of 50000 trading cycles have been discarded in order to avoid effects due to transients. a) Price history $p(t)$ over 10000 trading cycles. b) Log-returns $r(t) = \log(\frac{p(t)}{p(t-1)})$ with respect to the price history shown in a). c) Auto-covariance $C_x(\Delta t) = \frac{\sum_t (x(t) - \langle x(t) \rangle)(x(t+\Delta t) - \langle x(t+\Delta t) \rangle)}{\sum_t (x(t) - \langle x(t) \rangle)^2}$ determined on the basis of 50000 trading cycles. The dashed line shows the auto covariance for the absolute values of log-returns $x = |r(t)|$, the solid line shows the auto covariance for $x = r(t)$. d) Normalized distribution of log-returns $P(r)$ for $\Delta t = 1$ on a semilog plot. For comparison, the solid line shows the standard normal distribution. e) Normalized distribution of log-returns $P(r)$ for $\Delta t = 1, 4, 9, 16, 25$. Curves are shifted in vertical direction for the sake of clarity. f) Powerspectra (mean over 4 intervals of 10000 trading cycles).

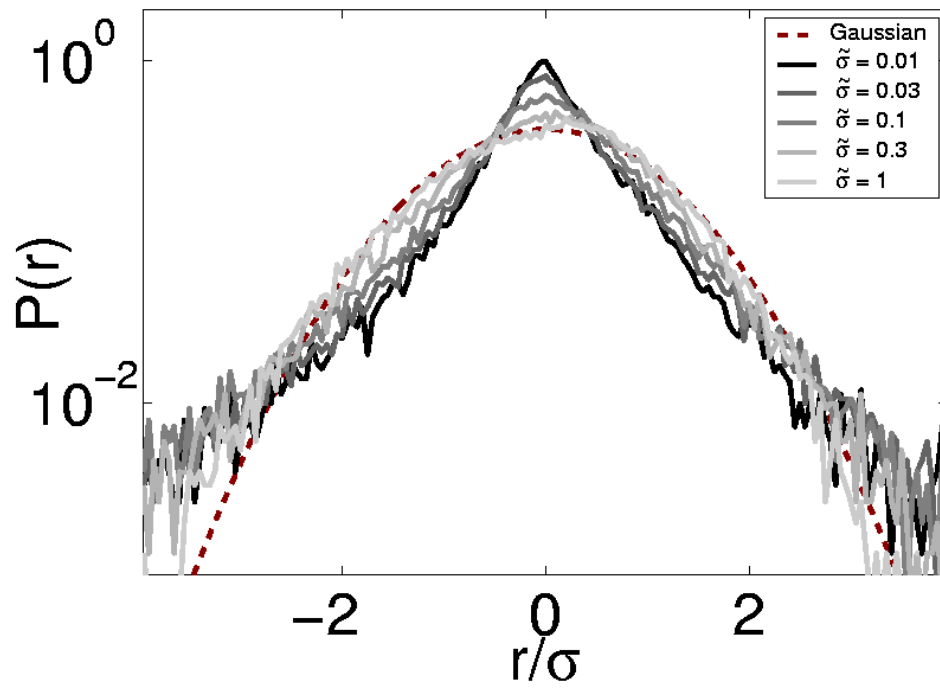


Fig. 4. Normalized return distribution calculated as in Fig 3 for different values of $\tilde{\sigma}$ ranging from 0.01 to 1.

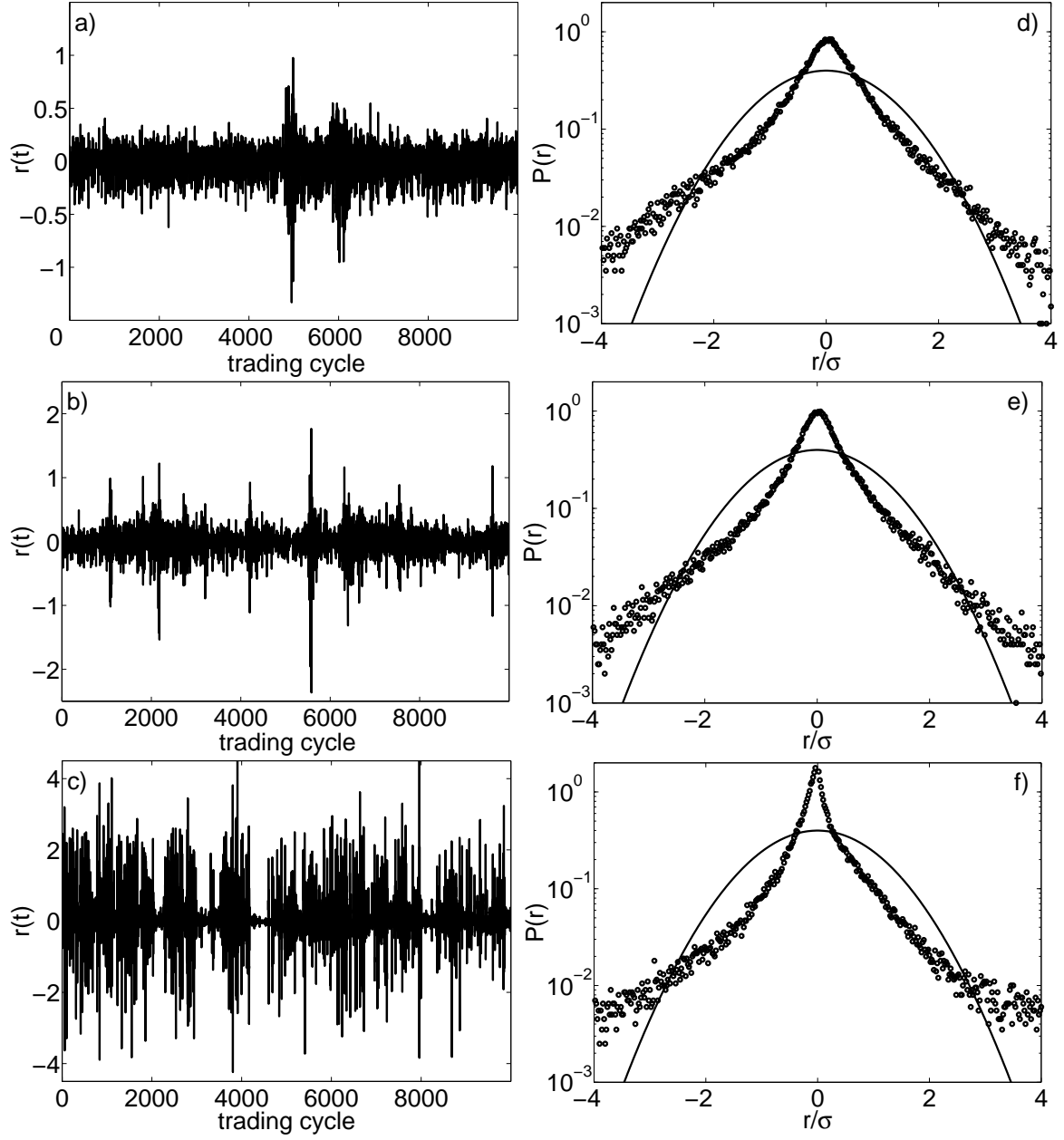


Fig. 5. Simulation results obtained from different evolutionary models with parameters $N = 1000$, $P = 3$, $S(0) = 1000$, $M(0) = 100000$, $\sigma = 1$. After each trading cycle the poorest(upper), a random(middle) or the richest(lower) agent is replaced by a new random one. Left and right plots are the same as in Fig.3 b) and d), respectively.